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Bianchi Type-VI_0 String Cosmological Model For Barotropic Fluid Distribution with Cosmological Term Λ

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Abstract: Bianchi type-VI_0 string cosmological model for barotropic fluid distribution with cosmological term Λ is investigated. To obtain the explicit solution of the model, we suppose expansion θ is propotional to the shear σ and Λ is proportional to R^{-3}, where R is scale factor. We have also discussed physical and geometrical characteristics of the cosmological model.
Keywords: Bianchi type-VI_0, barotropic fluid, cosmic string, cosmological term, perfect fluid.

I. INTRODUCTION

In recent years, the study of cosmological models with dark energy create much more interest due to the fact that our universe is undergoing an accelerated expansion which is driven by an exotic energy with large negative pressure named dark energy. Cosmologists have proposed many candidates for this dark energy, however, cosmological constant Λ is the simplest and most hypothetically attractive candidate of it with equation of state $\omega = -1$. According to recent observations, $\Lambda \approx 10^{-56} cm^{-2}$ while particle physics predicts that Λ is much greater than this value by factor of order 10^{120}. This inconsistency is called cosmological constant problem and the simplest way to resolve this problem is to consider cosmological termΛ. Bhoyar et al. [3], Pradhan et al. [10], Tyagi et al. [14] have studied cosmological models with cosmological term (Λ) in various contexts.

Due to the large scale distribution of galaxies in our cosmos, the matter distribution can be suitably described by perfect fluid. Bianchi Type-VI_0 cosmological models in different context have studied by different researchers viz. Amirhaschi [1], Deo et al. [4], Khadekar et al. [6, 7], Pradhan et al. [8, 9], Tyagi et al. [11, 13], Verma et al. [15]. String dust cosmological model for barotropic fluid distribution with vacuum energy density in FRW space-time is investigated by Bali and Bola [2]. Dewri [5] has studied magnetized anisotropic Bianchi type-VI cosmological model containing dark energy. Trivedi and Shrimali [12] have investigated barotropic Bianchi type-VI_0 cosmological model in general relativity.

In this paper, we have investigated Bianchi type-VI_0 string cosmological model for barotropic fluid distribution and cosmological termΛ. To obtain the explicit solution of the model, we suppose expansion θ is propotional to the shear σ and Λ is proportional to R^{-3}, where R is scale factor. We have also discussed physical and geometrical characteristics of the cosmological model.

II. METRIC AND FIELD EQUATION

The line element for Bianchi type-VI_0 space-time is considered as

$$ds^2 = -dt^2 + A^2dx^2 + e^{-2x}B^2dy^2 + e^{2x}C^2dz^2 \quad (1)$$

where A, B and C depends on cosmic time t only.

The Einstein's field equation in the geometrized unit (c=8πG=1) is given by

$$R_i^j - \frac{1}{2}Rg_i^j + \Lambda g_i^j = -T_i^j \quad (2)$$

where R_i^j is Ricci tensor and $R = g^{ij}R_{ij}$ is Ricci scalar.

The energy momentum tensor (T_i^j) for the cloud of strings in the presence of perfect fluid distribution is given by

$$T_i^j = (\rho + p)v_iv^j + pg_i^j - \lambda x_ix^j \quad (3)$$

Here ρ is proper energy density, p is pressure and λ is string tension density. Also x^i, the unit space like vector specifying the direction of strings and v^i, the unit time like vector satisfying the following conditions:

$$v_i v^i = -1 = -x_i x^i \ and \ \ v^i x_i = 0 \tag{4}$$

In a co-moving coordinate system, we have

$$v^i = (0,0,0,1); \quad x^i = \left(\frac{1}{A},0,0,0\right) \tag{5}$$

The Einstein's field equation (2) for metric (1) together with (3) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{1}{A^2} + \Lambda = \lambda - p \tag{6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} + \Lambda = -p \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} + \Lambda = -p \tag{8}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} + \Lambda = \rho \tag{9}$$

$$\left[\frac{B_4}{B} - \frac{C_4}{C}\right] = 0 \tag{10}$$

III. SOLUTION OF FIELD EQUATIONS

The field equations (6) - (10) are the organization of five equations with seven undetermined factors A, B, C, λ, ρ, p and Λ, so to obtain an exact solution of the present model we suppose expansion θ is proportional to shear σ, which leads to

$$A = B^n \tag{11}$$

and Λ is inversely proportional to R^3, which leads to

$$\Lambda = \frac{\alpha}{R^3} = \frac{\alpha}{ABC} \tag{12}$$

where α is constant of proportion.

Equation (10) leads to

$$\frac{B_4}{B} = \frac{C_4}{C} \tag{13}$$

On integrating equation (13), we get

$$B = Ck \tag{14}$$

where k is the integrating constant.

Now, without loss of generality, we suppose k=1, then equation (14) becomes

$$B = C \tag{15}$$

To get the deterministic solution, we also use the barotropic fluid condition,

i.e. $p = \gamma\rho$ (16)

Now, using conditions (11), (12), (15) and (16) in field equations (6) – (9), we obtain

$$\frac{(n+1)B_{44}}{B} + \frac{[n^2 + (2n+1)\gamma]B_4^2}{B^2} - \frac{(\gamma+1)}{B^{2n}} + \frac{\alpha(\gamma+1)}{B^{n+2}} = 0 \tag{17}$$

Now we consider the following cases:

A. Dust Fluid Model ($\gamma = 0$)

On putting $\gamma = 0$ in equation (17), we get

$$\frac{(n+1)B_{44}}{B} + \frac{n^2 B_4^2}{B^2} - \frac{1}{B^{2n}} + \frac{\alpha}{B^{n+2}} = 0 \tag{18}$$

Equation (18) leads to

$$2B_{44} + \frac{2n^2 B_4^2}{(n+1)B} = \frac{2}{(n+1)B^{2n-1}} - \frac{2\alpha}{(n+1)B^{n+1}} \tag{19}$$

On putting $B_4 = f(B)$ and $B_{44} = ff'$ in equation (19), we get

$$\frac{df^2}{dB} + \frac{2n^2 f^2}{(n+1)B} = \frac{2}{(n+1)B^{2n-1}} - \frac{2\alpha}{(n+1)B^{n+1}} \tag{20}$$

On integrating equation (20), we get

$$f^2 = \frac{1}{B^{2(n-1)}} - \frac{2\alpha}{n(n-1)B^n} + \frac{l}{B^{\frac{2n^2}{n+1}}} \tag{21}$$

where l is constant of integration.

From equation (21), we have

$$\int \frac{dB}{\sqrt{\frac{1}{B^{2(n-1)}} - \frac{2\alpha}{n(n-1)B^n} + \frac{l}{B^{\frac{2n^2}{n+1}}}}} = \int dt + M = t + M \tag{22}$$

where M is the integrating constant. Value of B can be obtained from equation (22).

Hence, by appropriate transformation of coordinates i.e. B=T, x=X, y=Y and z=Z, metric (1) becomes

$$ds^2 = -\frac{dT^2}{\left[\frac{1}{T^{2(n-1)}} - \frac{2\alpha}{n(n-1)T^n} + \frac{l}{T^{\frac{2n^2}{n+1}}}\right]} + T^{2n}dX^2 + e^{-2X}T^2dY^2 + e^{2X}T^2dZ^2 \tag{23}$$

B. Stiff Fluid Model ($\gamma = 1$)

On putting $\gamma = 1$ in equation (17), we get

$$\frac{(n+1)B_{44}}{B} + \frac{[n^2+2n+1]B_4^2}{B^2} - \frac{2}{B^{2n}} + \frac{2\alpha}{B^{n+2}} = 0 \tag{24}$$

Equation (24) leads to

$$2B_{44} + \frac{2(n+1)B_4^2}{B} = \frac{4}{(n+1)B^{2n-1}} - \frac{4\alpha}{(n+1)B^{n+1}} \tag{25}$$

On putting $B_4 = f(B)$ and $B_{44} = ff'$ in equation (25), we get

$$\frac{df^2}{dB} + \frac{2(n+1)f^2}{B} = \frac{4}{(n+1)B^{2n-1}} - \frac{4\alpha}{(n+1)B^{n+1}} \tag{26}$$

On integrating equation (26), we get

$$f^2 = \frac{1}{(n+1)B^{2(n-1)}} - \frac{4\alpha}{(n+1)(n+2)B^n} + \frac{L}{B^{2(n+1)}} \tag{27}$$

where L is constant of integration.

From equation (27), we have

$$\int \frac{dB}{\sqrt{\frac{1}{(n+1)B^{2(n-1)}} - \frac{4\alpha}{(n+1)(n+2)B^n} + \frac{L}{B^{2(n+1)}}}} = \int dt + M' = t + M' \tag{28}$$

where M' is the integrating constant. Value of B can be obtained from equation (28).

Hence, by appropriate transformation of coordinates i.e. B=T, x=X, y=Y and z=Z, metric (1) becomes

$$ds^2 = -\frac{dT^2}{\left[\frac{1}{(n+1)T^{2(n-1)}} - \frac{4\alpha}{(n+1)(n+2)T^n} + \frac{L}{T^{2(n+1)}}\right]} + T^{2n}dX^2 + e^{-2X}T^2dY^2 + e^{2X}T^2dZ^2 \tag{29}$$

C. Radiation Dominated Model ($\gamma = 1/3$)

On putting $\gamma = 1/3$ in equation (17), we get

$$\frac{(n+1)B_{44}}{B}+\frac{[3n^2+2n+1]B_4^2}{3B^2}-\frac{4}{3B^{2n}}+\frac{4\alpha}{3B^{n+2}}=0 \tag{30}$$

Equation (30) leads to

$$2B_{44}+\frac{2[3n^2+2n+1]B_4^2}{3(n+1)B}=\frac{8}{3(n+1)B^{2n-1}}-\frac{8\alpha}{3(n+1)B^{n+1}} \tag{31}$$

On putting $B_4=f(B)$ and $B_{44}=ff'$ in equation (31), we get

$$\frac{df^2}{dB}+\frac{2[3n^2+2n+1]f^2}{3(n+1)B}=\frac{8}{3(n+1)B^{2n-1}}-\frac{8\alpha}{3(n+1)B^{n+1}} \tag{32}$$

On integrating equation (32), we get

$$f^2=\frac{2}{(n+2)B^{2(n-1)}}-\frac{8\alpha}{[3n^2+n+2]B^n}+\frac{N}{B^{\frac{2(3n^2+2n+1)}{3(n+1)}}} \tag{33}$$

where N is constant of integration.

From equation (33), we have

$$\int\frac{dB}{\sqrt{\frac{2}{(n+2)B^{2(n-1)}}-\frac{8\alpha}{[3n^2+n+2]B^n}+\frac{N}{B^{\frac{2(3n^2+2n+1)}{3(n+1)}}}}}=\int dt+P=t+P \tag{34}$$

where P is the integrating constant. Value of B can be obtained from equation (34).

Hence, by appropriate transformation of coordinates i.e. B=T, x=X, y=Y and z=Z, metric (1) becomes

$$ds^2=-\frac{dT^2}{\left[\frac{2}{(n+2)T^{2(n-1)}}-\frac{8\alpha}{[3n^2+n+2]T^n}+\frac{N}{B^{\frac{2(3n^2+2n+1)}{3(n+1)}}}\right]}+T^{2n}dX^2+e^{-2X}T^2dY^2+e^{2X}T^2dZ^2 \tag{35}$$

IV. PHYSICAL AND GEOMETRICAL CHARACTERISTICS

For the model (23), energy density (ρ), string tension density (λ), pressure (p), expansion θ, and shear σ are given by

$$\rho=\frac{2n}{T^{2n}}+\frac{\alpha(n^2-5n-2)}{n(n-1)T^{n+1}}+\frac{l(2n+1)}{T^{\frac{2n^2+2n+1}{n+1}}} \tag{36}$$

$$\lambda=\frac{(n+2)\alpha}{nT^{n+2}}-\frac{2(n-2)}{T^{2n}}+\frac{l(n+1-2n^2)}{(n+1)T^{\frac{2n^2+2n+1}{n+1}}} \tag{37}$$

$$p=0 \tag{38}$$

$$\theta=(n+2)\left[\frac{1}{T^{2n}}-\frac{2\alpha}{n(n-1)T^{n+2}}+\frac{l}{T^{\frac{2n^2+2n+1}{n+1}}}\right]^{\frac{1}{2}} \tag{39}$$

$$\sigma=\frac{(n-1)}{\sqrt{3}}\left[\frac{1}{T^{2n}}-\frac{2\alpha}{n(n-1)T^{n+2}}+\frac{l}{T^{\frac{2n^2+2n+1}{n+1}}}\right]^{\frac{1}{2}} \tag{40}$$

For the model (29), energy density (ρ), string tension density (λ), pressure (p), expansion θ, and shear σ are given by

$$\rho=p=\frac{n}{(n+1)T^{2n}}+\frac{\alpha(n^2-5n-2)}{(n+1)(n+2)T^{n+2}}+\frac{L(2n+1)}{T^{2(n+2)}} \tag{41}$$

$$\lambda=\frac{2(n-1)\alpha}{(n+1)T^{n+2}}+\frac{4}{(n+1)T^{2n}} \tag{42}$$

$$\theta = (n+2)\left[\frac{1}{(n+1)T^{2n}} - \frac{4\alpha}{(n+1)(n+2)T^{n+2}} + \frac{L}{T^{2n+4}}\right]^{\frac{1}{2}} \quad (43)$$

$$\sigma = \frac{(n-1)}{\sqrt{3}}\left[\frac{1}{(n+1)T^{2n}} - \frac{4\alpha}{(n+1)(n+2)T^{n+2}} + \frac{L}{T^{2n+4}}\right]^{\frac{1}{2}} \quad (44)$$

For the model (35), energy density (ρ), string tension density (λ), pressure (p), expansion θ, and shear σ are given by

$$\rho = \frac{3n}{(n+2)T^{2n}} + \frac{3\alpha(n^2-5n-2)}{(3n^2+n+2)T^{n+2}} + \frac{N(2n+1)}{T^{\frac{2(3n^2+5n+4)}{3(n+1)}}} \quad (45)$$

$$\lambda = \frac{4(n+2)(n-1)\alpha}{(3n^2+n+2)T^{n+2}} - \frac{2(n-4)}{(n+2)T^{2n}} - \frac{2(n-1)(2n+1)N}{3(n+1)T^{\frac{2(3n^2+5n+4)}{3(n+1)}}} \quad (46)$$

$$p = \frac{n}{(n+2)T^{2n}} + \frac{\alpha(n^2-5n-2)}{(3n^2+n+2)T^{n+2}} + \frac{N(2n+1)}{3T^{\frac{2(3n^2+5n+4)}{3(n+1)}}} \quad (47)$$

$$\theta = (n+2)\left[\frac{2}{(n+2)T^{2n}} - \frac{8\alpha}{(3n^2+n+2)T^{n+2}} + \frac{N}{T^{\frac{2(3n^2+5n+4)}{3(n+1)}}}\right]^{\frac{1}{2}} \quad (48)$$

$$\sigma = \frac{(n-1)}{\sqrt{3}}\left[\frac{2}{(n+2)T^{2n}} - \frac{8\alpha}{(3n^2+n+2)T^{n+2}} + \frac{N}{T^{\frac{2(3n^2+5n+4)}{3(n+1)}}}\right]^{\frac{1}{2}} \quad (49)$$

Also, cosmological term Λ for the models (23), (29) and (35) is given by

$$\Lambda = \frac{\alpha}{T^{n+2}} \quad (50)$$

The magnitude of rotation ω is zero, i.e.

$$\omega = 0 \quad (51)$$

V. CONCLUSION

The models (23), (29) and (35) start expanding with big bang at $T = 0$. The expansion θ is decreasing function of cosmic time T for n>0 and approaches to zero as $T \to \infty$ and also stops at $n = -2$. Since $T \to \infty, \frac{\sigma}{\theta} = \frac{n-1}{\sqrt{3}(n+2)} \neq 0$, therefore the model does not approach isotropy for large values of T, however it is isotropized for $n = 1$.

Cosmological term Λ for these models are found to be decreasing function of time T for n>-2 and approaches to zero at late time, which is in agreement with present astronomical observations. A point type singularity is observed as $T \to 0$, $g_{11} \to 0$, $g_{22} \to 0$, $g_{33} \to 0$ for $n > 0$.

Also, we can observe that energy density (ρ), string tension density (λ) and pressure (p) for these models are decreasing function of time T for n>0 and approaches to zero at $T \to \infty$.

Hence, in general, the models represent expanding, shearing and non-rotating universe.

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